Chapter 14 Differentiation 2

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1. Variables *x* and *y* are related by the equation $y = \frac{lnx}{e^x}$.

   a. Show that $\frac{dy}{dx} = \frac{1-x\,lnx}{xe^x}$.

   [4]

   b. Hence find the approximate change in *y* as *x* increases from 2 to 2 + *h*, where *h* is small.

   [2]

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2. The number, *B*, of a certain type of bacteria at time *t* days can be described by $B = 200e^{2t} + 800e^{-2t}$.

   a. Find the value of *B* when $t = 0$.

   [1]

   b. At the instant when $\frac{dB}{dt} = 1200$, show that $e^{4t} - 3e^{2t} - 4 = 0$.

   [3]

   c. Using the substitution $u = e^{2t}$, or otherwise, solve $e^{4t} - 3e^{2t} - 4 = 0$.

   [2]

3. It is given that $y = \frac{ln(2x^3+5)}{x-1}$ for $x > 1$.

a. Find the value of $\frac{dy}{dx}$ when *x* = 2. You must show all your working.

[4]

b. Find the approximate change in *y* as *x* increases from 2 to 2 + *p* , where *p* is small.

[1]

4. $f: x \to e^{3x}$ for $x \in \mathbb{R}$

$g: x \to 2x^2 + 1$ for $x \geq 0$

Solve $f'(x) = 6g''(x)$, giving your answer in the form $ln\ a$, where *a* is an integer.

[3]

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5. Two variables *x* and *y* are such that $y = \frac{lnx}{x^3}$ for $x > 0$.

a. Show that $\frac{dy}{dx} = \frac{1-3\ lnx}{x^4}$.

[3]

b. Hence find the approximate change in *y* as *x* increases from e to e + *h*, where *h* is small.

[2]

6. The variables *x*, *y* and *u* are such that $y = tan\,u$ and $x = u^3 + 1$.

a. State the rate of change of *y* with respect to *u*.

[1]

b. Hence find the rate of change of *y* with respect to *x*, giving your answer in terms of *x*.

[4]

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7. Given that $y = \frac{\sin x}{\ln x^2}$, find an expression for $\frac{dy}{dx}$.

[4]

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8. Differentiate $\tan 3x \cos \frac{x}{2}$ with respect to x.

[4]

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9. It is given that $y = \frac{ln(4x^2+1)}{2x-3}$.

a. Find $\frac{dy}{dx}$.

[3]

b. Find the approximate change in *y* as *x* increases from 2 to 2+*h*, where *h* is small.

[2]

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10. It is given that $y = (1 + e^{x^2})(x + 5)$.

a. Find $\frac{dy}{dx}$.

[3]

b. Find the approximate change in *y* as *x* increases from *0.5* to *0.5*+*p,* where *p* is small.

[2]

c. Given that *y* is increasing at a rate of 2 units per second when *x* = 0.5, find the corresponding rate of change in *x*.

[2]

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11. The equation of a curve is given by $y = xe^{-2x}$.

a. Find $\frac{dy}{dx}$.

[3]

b. Find the exact coordinates of the stationary point on the curve $y = xe^{-2x}$.

[2]

c. Find, in terms of e, the equation of the tangent to the curve $y = xe^{-2x}$ at the point $(1, \frac{1}{e^2})$.

[2]

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12. Given that $y = 2sin\,3x + cos\,3x$, show that $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = ksin\,3x$, where k is a constant to be determined.

[5]